MSE160 [Winter 2023] – Problem Set # 1

Anusha Fatima Alam
Student No. 1009056539
UTORid: alamanus
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Problem 1 –

1. Short Answer (10 pts)

a) Explain how tempered glass resists stresses. Use a diagram. (3)
b) Define Poisson's ratio. What does it tell us? (2)
c) Define Hooke's law and the engineering stress-strain relationship. How is Hooke's law related to stress? How is it different? (2)

Q1. Part A:

Tempered Glass is produced from thermal and chemical treatments. During these treatments, as the glass cools, the center is subject to tension and the outer surface is under compression (see figure 1.1, the diagram of the stress distribution). This is because the outer surface will cool faster, whilst the inner cross-section will cool slower.

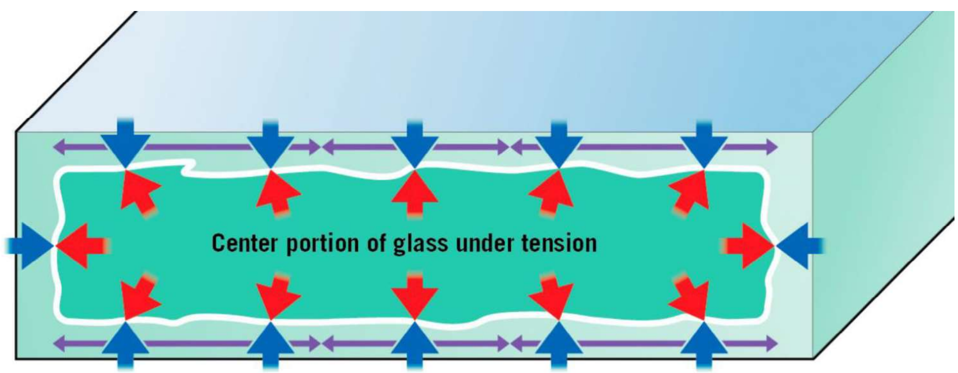


Figure 1.1 Tempered Glass Stress Balance

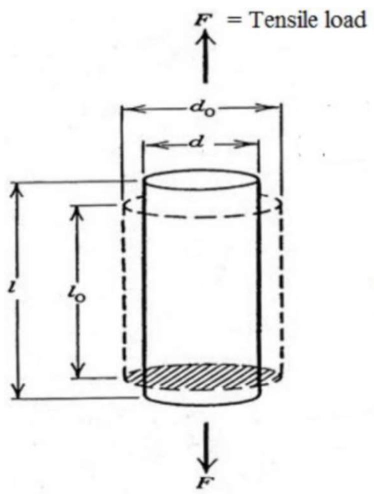
Due to the residual compressive stress on the surface, tempered glass has greater strength however it can easily shatter into small pieces when this outer surface is broken (i.e., fragmentation, when it is unable to carry the tensile stresses). Essentially it will shatter into granules rather than shards.

This high residual compressive stress on the surface is accompanied by a residual tensile stress in the center which creates a balance where risk of fragmentation is minimized and hence the apparent strength is higher because the center is sealed from the surroundings. Therefore, the strength is time dependent and sufficient to sustain high permanent tensile stresses.

This property of thermal glass makes it 4 times stronger than regular glass and helps it resist stresses.

(Question 1, Part B & C response continued on the following page)

Q1. Part B:



Poisson's ratio is a unitless ratio between the transversal (Lateral strain) and the axial (linear strain) which can be described as the as ratio of change of the width with respect to the original width of a material to the change in length with respect to the original length of the material, because of strain. It can be computed using the following formula:

$$Poisson's\ Ratio = \frac{Lateral\ Strain}{Linear\ Strain} = -\frac{d\epsilon_{trans}}{d\epsilon_{axial}}$$

$$Linear\ Strain = \frac{\Delta L}{L_0} = \frac{l - l_0}{l_0}$$

$$Lateral\ Strain = \frac{\Delta d}{d_0} = \frac{d - d_0}{d_0}$$

It essentially summarizes the ratio between the transversal contraction of the material (laterally) and the longitudinal extension in the direction of stretching force when it is subject to a tensile load.

Q1. Part C

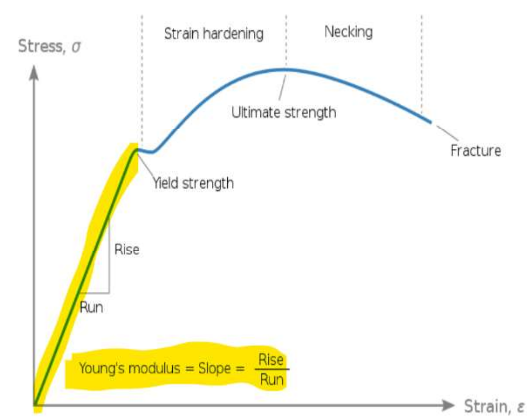
Hooke's Law – Hooke's Law is a mathematical relationship which states that the force applied to an object to extend or compress (say, a spring for example) by some distance (before it reaches its elastic limit) is directly proportional to its negative displacement. It can be described by the following expressions: $\mathrm{F} = -\mathrm{k}x$, where k is the constant of proportionality.

The negative sign denotes that the direction of the applied force is opposite to the direction of extension/displacement.

Engineering Stress-Strain – The engineering stress-strain relationship is given by the equation:

$$\sigma = \mathrm{E} \cdot \varepsilon$$

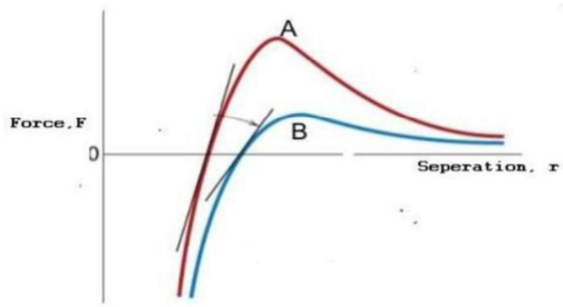
where σ, is the engineering stress [MPa], E is the young's modulus [MPa], and ε, is the engineering strain. This equation holds for under elastic deformation (before any permanent deformations occur). After the object reaches its tensile strength, permanent (plastic) deformations occur, and this proportionality no longer holds. This relationship can be determined by gradually applying tensile load to a testing coupon and observing the elongation and deformation. This relationship can be observed on the stress-strain curve (see diagram on the right).



How is Hooke's law similar/different to stress? Hooke's law implicitly describes that the material strain is proportional to the material stress within elastic limits only, this is because the force is proportional to the negative displacement. They are different because Hooke's law describes the relationship between force and displacement whilst stress is the relationship between force with respect to the cross-sectional area of the object.

Problem 2 -

The following questions concern two hypothetical materials, A and B, with these curves showing the net interatomic forces as a function of interatomic separation. Which material will have a higher modulus of elasticity (Young's modulus), and why? (3 pts)



Directly Proportional Relationship:

Using the proportional relationship between the young's modulus and the slope/gradient of the (force and separation distance) curve:

$$E \propto \left.\frac{dF}{dr}\right|_{r = r_0}$$

where r_0 is the distance where $F_{net} = 0$.

Answer: Material A

Conclusion/Explanation: From the graph, it can be observed that material A, has gradient of larger magnitude than material B. This is because the slope of curve A is steeper in comparison to the slope of curve B. Essentially the young's modulus is directly related to the interatomic bonding energies. Therefore strong bonding energies (like for Material A) will have a higher young's modulus. Due to the proportionality relationship outlined above, Material A will have a higher young's modulus. Material B will have a lower young's modulus.

Problem 3 –

A bar is at a length of 31.5cm when 200kN of force is applied in tension. The radius of the bar is 15mm. If the strain experienced by the bar is 0.015, what is the young's modulus for this material? (3 pts)

$$F = 200\ [kN] = 200{,}000\ [N]$$

$$L = 31.5\ [cm] = 315\ [mm]$$

The cross-sectional area of the circular bar is:

$$A = \pi r^2 = \pi(15\ [mm])^2 = 225\pi\ [mm^2] \approx 707\ [mm^2]\ 3\ s.f.$$

The engineering stress experienced by the bar is:

$$\sigma\ [MPa] = \frac{F\ [N]}{A\ [mm^2]} = \frac{200{,}000\ [N]}{225\pi\ [mm^2]} \approx 283\ [MPa]\ 3\ s.f.$$

The Young's Modulus can be calculated using the stress-strain relationship:

$$E\ [MPa] = \frac{\sigma\ (stress)}{\varepsilon\ (strain)} = \frac{200{,}000\ [N]}{225\pi\ [mm^2] \cdot 0.015} \approx 18862.8\ MPa$$

$$\boxed{\therefore\ E\ (Young's\ Modulus) = 18.9\ GPa\ *}$$

$*\ Quoted\ according\ 3\ Significant\ Figures$

Problem 4 –

For a brass alloy, the stress at which plastic deformation begins is 345 MPa, and the modulus of elasticity is 103 GPa. You are given 0.5m long hollow brass cylinder with an internal external diameter of 2 cm and an internal diameter of 1.8cm, which will be required to support a large chandelier. (5 pts)

a. *What is the maximum load that can be supported without plastic deformation? (3 points)*
b. *What is the maximum length to which it can be stretched without causing plastic deformation? (2 points)*

Q4. Part A:

The yield strength is the stress which marks the beginning of plastic deformation.

$$\sigma_{yield} = 345\ [MPa]$$

To determine the maximum load to support without plastic deformation, the following must hold:

$$\sigma < \sigma_{yield}$$

$$\sigma < 345\ [MPa]$$

Substituting the formula for σ_{yield} into the inequality above:

$$[\sigma =]\ \frac{F}{A} < 345\ [MPa]$$

The cross-sectional area of the hollow-cylindrical brass alloy can be determined by:

$$A = \pi(r_{outer})^2 - \pi(r_{inner})^2 = \pi\left((20\ [mm])^2 - (18\ [mm])^2\right) = 76\pi \approx 239\ mm^2$$

The inequality can be rewritten in terms of F to determine the minimum F w/o plastic deformation:

$$F < 345 \cdot A\ (= 345 \cdot 76\pi = 26229\,\pi = 82.37\ [kN])$$

$$\boxed{F_{min} = 82.3\ [kN] *}$$

$*$ *Quoted according 3 Significant Figures (rounded down, because minimum force is below 82.4 kN)*

Q4. Part B:

Since, the stress needs to be below the yield stress to ensure that plastic deformation doesn't occur:

$$\sigma < \sigma_{yield} \quad \rightarrow \quad \sigma < 345\ [MPa]$$

$$\sigma = E \cdot \varepsilon = 103\,000[MPa] \cdot \varepsilon < 345\ [MPa]$$

Rewriting this equation in terms of original length, $l_0 = 500\ mm$, and L_{max}:

$$\left[\varepsilon = \frac{\Delta L}{L_0} = \frac{l - l_0}{l_0}\right] = \frac{L_{max} - 500[mm]}{500[mm]} < \frac{345[MPa]}{103{,}000[MPa]}\,[= 3.35 \times 10^{-3}]$$

$$L_{max} < \frac{345[MPa] \cdot 500[mm]}{103{,}000[MPa]} + 500 = 501.67\ [mm]$$

Therefore, the maximum length it can reach before plastic deformation is $\boxed{\text{501 mm.}}$

Problem 5 –

A beam (dimensions 3 cm x 3 cm x 10 cm) is placed under a tensile stress of 5000 N along its length. Measuring with calipers while the load is applied, it was found that the dimensions of the beam's cross section were now 2.99 cm x 2.99 cm. (5 pts)

a. *If the Poisson's ratio is 0.28, what is the Young's modulus of this unknown material? (3)*
b. *What is the shear modulus of this material? (2)*

Q5. Part A:

The tensile stress observed by the beam can be determined by:

$$\sigma = \frac{F}{A} = \frac{5000[N]}{30[mm] \cdot 30[mm]} = \frac{50}{9}\left[\frac{N}{mm^2}\right] \approx 5.56\ [MPa]$$

Poisson's ratio can be described as the following ratio:

$$Poisson's\ Ratio\ = \frac{Lateral\ Strain}{Linear\ Strain} = -\frac{d\epsilon_{trans}}{d\epsilon_{axial}} = 0.28$$

where the linear and lateral strain can be defined by:

$$Linear\ Strain\ = \frac{\Delta L}{L_0} = \frac{l - l_0}{l_0} = \varepsilon$$

$$Lateral\ Strain\ = \frac{\Delta d}{d_0} = \frac{d - d_0}{d_0} = \frac{29.9\ [mm] - 30\ [mm]}{30\ [mm]} = -\frac{1}{300}$$

Using the lateral strain and the Poisson's ratio, linear strain can be determined.

$$Linear\ Strain\ = \frac{Lateral\ Strain}{Poisson\ Ratio} = \frac{(-\frac{1}{300})}{0.28} = -\frac{1}{84} \approx -0.012$$

Using the stress-strain relationship, young's modulus can be calculated:

$$\boxed{E\ = \frac{\sigma}{\varepsilon} = \frac{\frac{50}{9}\ [MPa]}{0.012} = 466.7 \approx 467\ MPa\ (3.\mathrm{s.f})}$$

Q5. Part B:

According to the relationship between the young's modulus and the shear modulus:

$$E\ = 2G(1 + \upsilon)$$

where E, is the young's modulus; G, is the shear modulus; and V, is the Poisson's ratio.

Rearranging this equation for G, the shear modulus:

$$\boxed{\therefore G\ = \frac{E}{2\ +\ 2\nu} = \frac{463}{2.56} = 182.3\ [MPa]}$$

Problem 6 –

A three-point bending test is performed on a glass specimen having a rectangular cross section of height d = 5 mm and width b = 10 mm. The distance between support points is 45 mm. Compute the flexural strength if the load at fracture is 290 N. (2)

The flexural strength from three-point bending can be computed using the formula:

$$\sigma_f = \frac{3FL}{2bd^2} = \frac{3 \cdot 290\,[N] \cdot 45\,[mm]}{2 \cdot 10\,[mm] \cdot (5\,[mm])^2} = 78.3\,[MPa]$$

$$\boxed{\therefore \sigma_f = 78.3\,[MPa]}$$

Problem 7 –

A metal component made of unobtanium must be fabricated from a single crystal. This component will have a final volume of 12.3 cubic metres. Unobtanium has a face-centred cubic crystal structure, the atoms have a radius of 1.35 Å, and a molar mass of 75.5 g/mol. Determine the final mass of this component. (3 points)

Rewriting the atomic radius in SI units:

$$\text{Atomic radius (r)} = 1.35\text{ Å} = 1.35 \times 10^{-10}\,[m]$$

Determining the atomic volume of spheres in the crystal using the atomic packing factor:

$$APF_{FCC} = \frac{Volume_{Spheres}}{Volume_{Unit\ Cell}} = 0.74$$

$$Volume_{Spheres} = 0.74 \cdot Volume_{Unit\ Cell} = 0.74 \cdot 12.3\,[m^3] = 9.102\,[m^3]$$

The volume of a single atom of Unobtanium using the atomic radius is given by:

$$Volume_{1\ sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.35 \times 10^{-10})^3 = 1.0305 \times 10^{-29}\,[m^3]$$

Calculating the maximum number of atoms within the sample of crystal structure:

$$\#\ of\ atoms = \frac{Volume_{Spheres}}{Volume_{1\ sphere}} = \frac{9.102}{1.0305 \times 10^{-29}} = 8.8315 \times 10^{29}$$

Computing the number of moles using Avogadro's Number:

$$n(mols) = \frac{\#\ of\ atoms}{N_A} = \frac{8.8315 \times 10^{29}}{6.023 \times 10^{23}} = 1446337\,[mols]$$

Determining the final mass of the sample using the molar mass:

$$m\,[g] = n\,[mols] \cdot M_r\,[g/mol] = 1446337 * 75.5\,[g]$$

$$\boxed{\therefore Final\ Mass = 1.107 \times 10^8\ g}$$